ON THE NUMBER SPECTRUM OF MUON BUNDLES UNDERGROUND*

A simple model of hadronic cascade in the atmosphere is suggested to calculate expected number of muons above a given energy threshold as a function of primary energy. The result shows the power law number spectrum for high multiplicities of muons ($m > 3$) with the exponent $\alpha = -(3.0 \div 3.3)$ not depending on depth and primary composition. For $m < 3$ the number spectrum is affected by atomic number of primaries, by depth, and also by the size of the detector if the latter is not big enough.

Introduction

The application of big underground detectors for investigation of muon bundles promises an interesting information concerning high energy phenomena in cosmic rays. If the size of the detector a is bigger than the lateral spread of the bundle r_μ, then the total number of muons associated with an event can be measured, not a small and unknown portion like in most cases formerly. The Utah detector and now KGF and Baksan detectors partly approach the condition $a >> r_\mu$. In this case one can expect a strong correlation between the number of muons and primary energy, like in EAS. Also there is a hope to obtain a useful information on pion and kaon production, transverse momenta and primary composition.

The calculation of muon bundle characteristics, assuming as known all elementary processes involved, seems to be a very complicated procedure and generally can be performed only by Monte-Carlo method. Nevertheless, the analytical approach based on a simple model could be useful to look at the principal features of the phenomenon in a qualitative way.

That kind of analysis was made by Koshiba and Totsuka [1] using a postulate that multiplicity distribution of muon parents for a given energy threshold obeys a power law.

In the present paper a modification of above mentioned approach is suggested which differs from [1] by the following:

1) A definite model of hadronic cascade is introduced.

2) In this model the continuous quantity - the expected number of muon parents is used. Correspondingly, the binominal distribution in [1] is replaced by the Poissonian.

3) The dependence on primary atomic number A is estimated.

4) The case $a << r_\mu$ and $a \sim r_\mu$ occurred to be more complicated than [1].

A model of hadronic cascade. ("Quasi scaling model")

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* *Proc. of 16th Intern. Cosm. Ray Conf., Kyoto, 1979, vol.* ***10****, p. 192.*

To describe the number of pions generated in the cascade from primary nucleon of energy E_0 , it is naturally to satisfy two conditions :

1) For $E_\pi < E_0$ scaling in fragmentation region demands $N_\pi(>E_\pi) = F(E_\pi/E_0)$;

2) For $E_\pi << E_0$ and E_0 big enough one can assume, like in EAS, $N_\pi \approx E_0^\delta$, where δ = 0.7 - 0.8. The power law in conventional model can be realized only if pions take part in the cascade. Then

$$\delta = 1 + \frac{\ln \frac{2}{3}}{\ln \nu} ,$$

where ν is an effective multiplicity in νN-interaction, 2/3 is the ratio of charged pions to all pions. Thus deviation of δ from unity depends on leakage of hadronic cascade energy into neutral pions.

To fit both conditions the "quasi scaling" function $F(E_\pi/E_0)$ for $E_\pi/E_0 << 1$ should be

$$F \approx \left(\frac{E_\pi}{E_0}\right)^{-\delta} .$$

The simplest particular variant is $\displaystyle \frac{dN}{dE_\pi} \approx \left(\frac{E_\pi}{E_0}\right)^{-\delta-1}$.

Then the energy spectrum and number of muons are:

$$\left.\begin{aligned} &\frac{dN}{dE} = \frac{C}{E_0}\cdot\left(\frac{E}{E_0}\right)^{-\delta-1}\cdot\frac{B}{B+E}; \\ &N(>E_\mu) = C\cdot y^\delta \int_{E_\mu/B}^{y} \frac{x^{-\delta-1}}{(1+x)}dx, \end{aligned}\right\} \qquad (1)$$

where $B/(B+E)$ is the effective decay probability of pions and kaons, $y=E_0/B$, and E_μ is the muon threshold energy. In this particular scheme the constant C can be obtained by direct comparison of single muon spectrum and primary spectrum. Below for numerical estimates C = 0.1 and B = 100 GeV will be used (vertical direction, pions only).

Muon number spectrum $J(m,E_\mu)$ when $a >> r_\mu$

Neglecting specific fluctuations in cascade development let us take the probability to observe m muons, when expected value is $N(y)$, as a Poisson distribution. Then

$$J(m,E_\mu) = S \int_{E_\mu/B}^{\infty} e^{-N(y)} \frac{[N(y)]^m}{m!} f_1(y)\, dy, \qquad (2)$$

where S is the area of a detector.

N(y) is taken from (1) and $f_1(y)$ is primary spectrum;

$$f_1(y) = D\, B^{-\gamma} y^{-\gamma-1} \qquad (3)$$

The solution can be simplified if instead of (1) one takes its asymptotic approximation

$$N(>E_\mu) = C \cdot y^\delta \cdot f_\delta\left(\frac{E_\mu}{B}\right); \quad f_\delta = \int_{E_\mu/B}^{\infty} \frac{x^{-\delta-1}}{(1+x)} dx \qquad (4)$$

and change variable in (2)

$$\left.\begin{aligned} J(m,E_\mu) &= S\int_{N_0}^{\infty} e^{-N} \frac{N^m}{m!} f_2(N)dN; \\ f_2(N) &= \frac{1}{\delta} \cdot D \cdot B^{-\gamma} (Cf_\delta)^{\frac{\gamma}{\delta}} \cdot N^{-\frac{\gamma}{\delta}-1}. \end{aligned}\right\} \qquad (5)$$

In this approximation the real behaviour of f_2 , when $N \to 0$ is replaced by a cut-off of the power spectrum if $N < N_0$ *) .

To determine N_0 let us require that the total flux of muons through *S* per sec from (5) should be the same as from (1) and (3).

$$K(E_\mu) = \sum_{m=1}^{\infty} m \cdot J(m,E_\mu) = S\int_{N_0}^{\infty} N \cdot f_2(N)dN =$$

$$= S \cdot D \cdot B^{-\gamma} \int_{E_\mu/B}^{\infty} y^{-\gamma-1} dy C y^\delta \int_{E_\mu/B}^{y} \frac{x^{-\delta-1}}{(1+x)} dx.$$

Performing the integration we obtain:

$$N_0\left(\frac{E_\mu}{B}\right) = C \cdot f_\delta^{\frac{\gamma}{\gamma-\delta}} \cdot f_\gamma^{\frac{\delta}{\delta-\gamma}}; \quad f_\gamma = \int_{E_\mu/B}^{\infty} \frac{x^{-\gamma-1}}{(1+x)} dx. \qquad (6)$$

Dividing *J* by the total muon flux *K* we have finally:

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*) This "N_0-approximation” in principle shifts some events from m = 1 to m = 2 thus overestimating the probability of pairs, which becomes significant for N_0 > 0.1.

$$\frac{J(m,E_\mu)}{K(E_\mu)}=\frac{\gamma-\delta}{\delta}N_0^{\frac{\gamma-\delta}{\delta}}\,W_m(N_0);\,W_m(N_0)=\int_{N_0}^{\infty}e^{-N}\frac{N^{m-\frac{\gamma}{\delta}-1}}{m!}dN. \qquad (7)$$

Thus, the number spectrum depends on two parameters: γ/δ and N_0 .

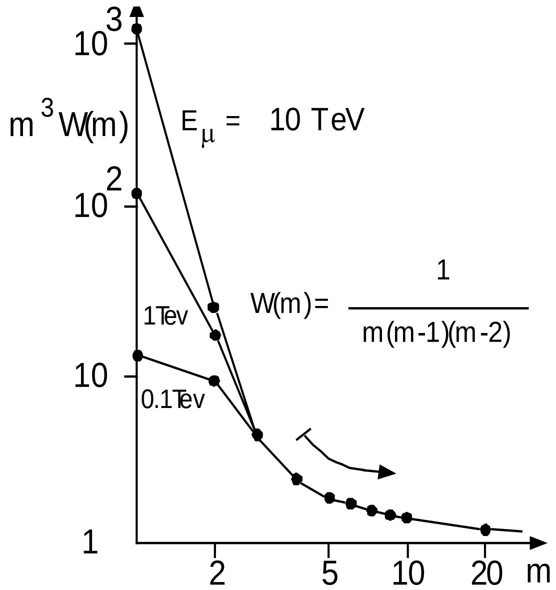

Fig. 1

For m > 3 the shape of the spectrum does not practically depend on N_0 asympotically following a power law:

$$\sim m^{\frac{\gamma}{\delta}-1}.$$

In this case there is a strong correlation between m and N, also between m and E_0 . For m = 1 and m = 2 a broad interval of primary energies is involved. The ratio of high multiplicity rate to the total one has a linear dependence on N_0 decreasing when E_μ increases.

Now as a numerical example let us take $\gamma/\delta = 2$ (also C = 0.1 ; B = 10^2 GeV/c). In this case:

$$\frac{J(m,E_\mu)}{K(E_\mu)}=N_0(E_\mu)\cdot W_m(N_0); \quad W_m=\int_{N_0}^{\infty}e^{-N}\frac{N^{m-3}}{m!}dN. \qquad (8)$$

Fig. 1 shows the deformation of the power law due to Poisson fluctuations and enhancement of $J(1)/K$ and $J(2)/K$ when E_μ increases.

Number spectrum from primaries with atomic number *A*

Supposing superposition model the previous result can be easily generalized to the case when primaries consist of particles with definite A. The only change is, that the parameter N_0 in formulae (7) and (8) should be replaced by a new one:

$$\tilde{N}_0 = A \cdot N_0. \qquad (9)$$

Fig. 2 shows the *A*-dependence of the ratio of multiple rate to total muon flux. For big *m* there is a linear rise with *A*. It should be specified that N_0 -approximation is correct only if $N_0 << 1$, also $\tilde{N}_0 << 1$, which is not valid when $E_\mu/A < B$. This affects the results for $m = 2$, (1), and slightly $m = 3$. So the absolute value of $J(m)/K$ for $m > 3$ in principle can be used as a best measure of average atomic number of primaries.

Muon number spectrum when $a << r_\mu$ and $a \sim r_\mu$

This case (a practical one) is more complicated for calculations and obviously less informative, because the lateral distribitlon of muons should be assumed. It is natural to expect, that the maximum size of the bundle

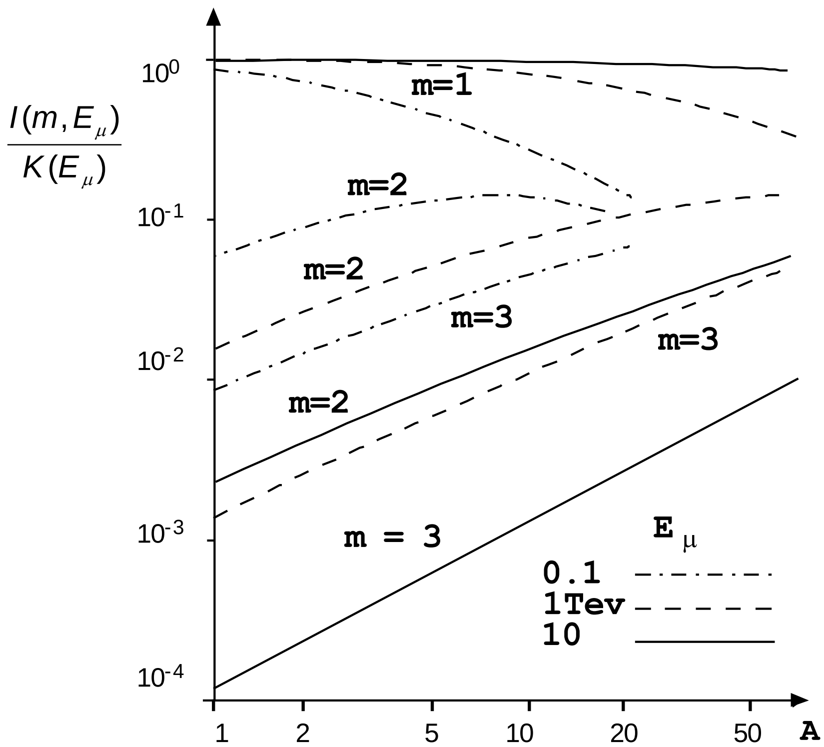

Fig. 2

$r_\mu \sim 1/E_\mu$, but there should be also a structure of a smaller size because of the presence of muons with $E >> E_\mu$ in the bundle. The existence of the pole in lateral structure is proved also by decoherence curve from [2,3] which seems to have no flattening at small distances.

Let us take the lateral distribution of muons with a given energy *E* in the form

$$\varphi(x) = \frac{1}{2\pi r^2} e^{-\frac{x}{r}}; \quad r = r_\mu \frac{E_\mu}{E}; \quad r_\mu = \frac{Hp_1}{E_\mu}, \qquad (10)$$

where *x* is the distance from the axis; *H* is effective height of production; $p_1 = \frac{1}{3}\bar{p}_t$ ($\bar{p}_t$ is mean transverse momentum of pions).

lntegrating $\varphi(x)$ over the muon energy spectrum (1) and normalizing to one muon we get lateral distribution function of the bundle:

$$\psi(x)=\frac{1}{2\pi r_\mu^2}\left(\frac{E_\mu}{B}\right)^{-2}\cdot\frac{1}{f_\delta\left(\frac{E_\mu}{B}\right)}\cdot\int_{E_\mu/B}^{\infty}\frac{\xi^{1-\delta}\,e^{-\frac{x}{r_\mu}\cdot\frac{B}{E_\mu}\xi}}{1+\xi}d\xi\,, \qquad (11)$$

where as in "N_0-approximation" the upper limit of the integral $\frac{E_0}{B}\to\infty$. This approximation makes $\psi(x)$ independent of E_0 and N. Then the part of the bundle $\Delta(\vec{\rho})=\frac{N'}{N}$ which happens to be inside detector is obtained by integrating over detector area:

$$\Delta(\vec{\rho})=\iint_S\psi\left(\left|\vec{\rho}-\vec{b}\right|\right)dS\,, \qquad (12)$$

where $\vec{\rho}$ corresponds to the axis of the bundle, $\vec{b}$ to the point inside detector.

Now we get the new density spectrum $S\cdot f_2'(N')$ simply replacing in (5) N by $\frac{N'}{\Delta(\rho)}$ and integrating over ρ:

$$Sf_2'(N')=f_2(N')\int_0^{\infty}2\pi\rho\,d\rho\left[\Delta(\vec{\rho})\right]^{\frac{\gamma}{\delta}}=S'f_2(N')\,. \qquad (13)$$

Here S is the area of a detector, $S'=\int 2\pi\rho\Delta^{\frac{\gamma}{\delta}}d\rho$ some ficticious area $S'<S$. We have the same power law approximations as (5), (6) with only difference that N_0 should be replaced by a smaller quantity N_0' :

$$N_0'=\left(\frac{S'}{S}\right)^{\frac{\delta}{\gamma-\delta}}\cdot N_0. \qquad (14)$$

To get numerical results it is necessary to perform integrations (11), (12), and (13). The last ones will be simplier if detector is a circle. Fig. 3 shows an example for a circle detector, radius a = 1 m. Lateral spread parameter is chosen $r_\mu E_\mu$ = 1 m x 1 TeV which corresponds to p_1 = 0.1 GeV/c and H = 10 km. Others parameters same as in Fig. 1. The relative rate of multiple events has a maximum at E_μ = 1.6 TeV which corresponds to $r_\mu\approx 0.7a$. The dotted lines represent the case $a \gg r_\mu$. The experimental points from [4] (S = 4 m^2) are also plotted. It would be prematurely to make conclusions from this comparison before adjusting constants involved, primary composition and analysis of other data. But one remark could be made. The authors of [4] interpreted the last point (the biggest depth) as to be lower than expected from extrapolation and thus confirming the proposed increase in lateral spread. One can see from fig. 3 that situation is rather opposite, and this particular point seems to be higher than expected. The fit could be made by rising the effective atomic number or prompt mechanism of muon production.

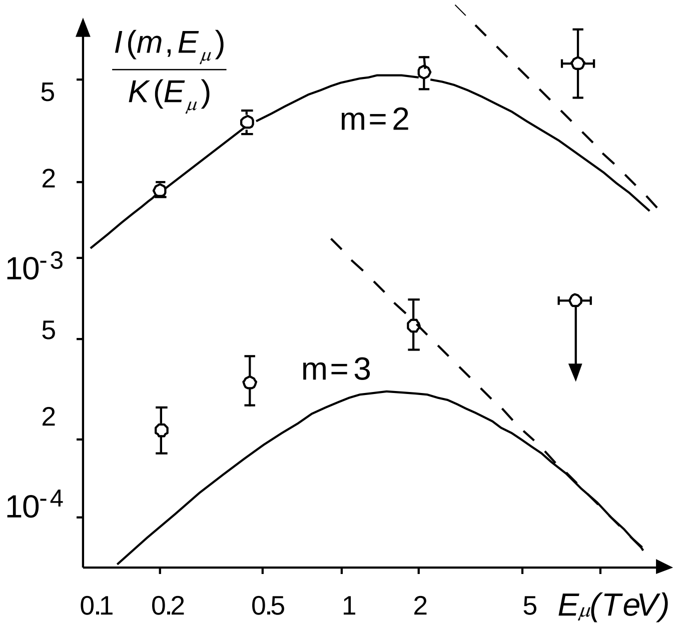

Fig. 3

Conclusions

Many features of muon bundles number spectrum obtained here are quite similar to that of Totsuka and Koshiba. The use of "quasi-scaling" model provides a more definite analysis of low multiplicity rate. Actually more realistic quasi-scaling function than (1) can be easily taken into account by adjusting constant C by Monte Carlo method or by fitting C to experimental data.

Also for m = 2 the calculations can be carried out with better validity directly using eq. (1), (2) without "N_0-approximation" .

In the case $a << r_\mu$ the pole in the muon lateral structure was found to be important. The "attraction of cores" to detector centre increases the ratio of multiple events significantly, for $a/r_\mu \sim 1$ by an order of magnitude.

Acknowledgments

The author wish to thank professor S. Miyake for kind hospitality during his stay at the Institute where this work was carried out. The stimulating discussion with professor M. Koshiba is appreciated.

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